

Total energy: $E_{tot} = \frac{1}{2} m_e v^2 + \frac{q_1 q_2}{4\pi\epsilon_0 r}$

Quantized angular momentum: $L = m_e v r = n \frac{h}{2\pi}$ Solve for v : $v = \frac{nh}{2\pi m_e r}$

Coulomb force law: $|F_{coulomb}| = \left| \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \right| = |m_e a|$ Rearrange: $\left| \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \right| = \left| \frac{m_e v^2}{r} \right|$

Insert v , solve for r : $r_n = \frac{\epsilon_0 n^2 h^2}{\pi m_e q_1 q_2} = \frac{\epsilon_0 n^2 h^2}{\pi m_e Z e^2}$

Plug r_n into v , solve for v_n : $v_n = \frac{Z e^2}{2\epsilon_0 n h}$

Plug r_n and v_n into E_{tot} , solve for E_n : $E_n = \frac{-Z^2 e^4 m_e}{8\epsilon_0^2 n^2 h^2}$