CH 301H Fall 2011

HW13 Key

1. a)

When the container is opened, gas from the room will flow into the container until an equilibrium pressure is reached.

b)

When the container is opened, gas from the container will flow into the room until an equilibrium pressure is reached

c) Based on the two scenarios above, in order to open the container safely, the pressure in the container needs to be less than the pressure of the room. Let's set it to 0.95 atm

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \qquad T_2 = \frac{P_2 T_1}{P_1} = \frac{(0.95\,atm)(293\,K)}{(1.47\,atm)}$$

$$\boxed{T_2 = 189\,K = -84°C}$$

2. a) The physical characteristics of a gas are P, V, and T. The property that is reported here, mass / volume, or density, is highly dependent on P, V, and T. Therefore, without knowing those values, that density information is useless.

b) $\rho = 6.234\ g/L$

$FW(H_2Te) = 129.6\ g/mol$

$P = 1.0\ atm$

$T = ?$

$PV = nRT \quad ; \quad n = \frac{m}{FW}$

$PV = \frac{mRT}{FW}$

$P = \frac{mRT}{V\,FW} = \frac{\rho RT}{FW}$

$T = \frac{P(FW)}{\rho R} = \frac{(1.0\ atm)(129.6\ g/mol)}{(6.234\ g/L)(8.21\times10^{-2}\ L\,atm/mol\,K)}$

$\boxed{T = 253\ K}$

3. $P = 1\times10^{-12}\ Torr\left(\frac{1\ atm}{760\ Torr}\right) = 1.32\times10^{-15}\ atm$

$V = 1.00\ cm^3\left(\frac{1\ m}{100\ cm}\right)^3\left(\frac{10\ dm}{1\ m}\right)^3 = 0.00100\ L$

$T = 298\ K$

$PV = nRT; \quad n = \frac{PV}{RT} = \frac{(1.32\times10^{-15}\ atm)(0.00100\ L)}{(8.206\times10^{-2}\ L\,atm/mol\,K)(298\ K)}$

$n = 5.4\times10^{-20}\ mol\ (6.022\times10^{23}\ atoms/mole)$

$V_m = \frac{V}{n} = \frac{0.00100\ L}{5.4\times10^{-20}\ mol} = \boxed{1.9\times10^{16}\ L/mol} \Rightarrow \boxed{3.3\times10^{4}\ \text{molecules in } 1\ cm^3}$

molar volume

4. $n = 1.0\,mol\ C_2H_4$

a) ideal: $V = 22.4L$
$T = 273K$

$$P = \frac{nRT}{V} = \frac{(1.0mol)(8.02\times10^{-2}\ Latm/mol\,K)(273K)}{22.4L}$$

$\underline{P = 1.0 atm}$

vdW: $P = \frac{nRT}{V-nb} - a\left(\frac{n}{V}\right)^2$ $a = 5.507\,L^2 atm/mol^2$
$b = 6.5\times10^{-2}\ L/mol$

$$P = \frac{(1.0mol)(8.02\times10^{-2}\ Latm/mol\,K)(273K)}{(22.4L) - (1.0mol)(6.51\times10^{-2}\ L/mol)} - (5.507\ L^2atm/mol)\left(\frac{1.0mol}{22.4L}\right)^2$$

$\underline{P = 1.0 atm}$ (identical within sig figs)

b) $T = 1000K$
$V = 0.100L$

ideal: $P = 8.2\times10^2$ atm
vdW: $P = 1.7\times10^3$ atm

The ideal gas law underestimates the pressure by more than a factor of 2. One conclusion is that for this molecule, the ideal gas law is a reasonable estimate at room temperature and pressure, but breaks down under extreme conditions.

5. $\rho(H_2(g)) = 1\text{molecule}/m^3$

$T = 10k$

$d = 1.15\times10^{-10}m$

$\sigma = \pi d^2 = (3.14)(1.15\times10^{-10}m)^2$

$\sigma = 4.15\times10^{-20}m^2$

$z_{coll} = N_D \sigma c_{rel}$ $\qquad c_{rel} = \sqrt{2}\, c_{rms}$

$\lambda = \frac{c_{rms}}{z_{coll}} = \frac{\cancel{c_{rms}}}{N_D \sigma \sqrt{2}\, \cancel{c_{rms}}} = \frac{1}{N_D \sigma \sqrt{2}}$

$\lambda = \frac{1}{(1\text{molecule}/m^3)(4.15\times10^{-20}m^2)\sqrt{2}}$

$\boxed{\lambda = 1.7\times10^{19}m}$

6. $Ar(g)$ $V = 1L$

$T = 298k$

$d(Ar) = 1.9\times10^{-10}m$

$\lambda = \frac{k_B T}{\sqrt{2}\sigma P}$

$P = \frac{k_B T}{\sqrt{2}\sigma\lambda}$

$V = 1L\left(\frac{1dm^3}{1L}\right)\left(\frac{1m}{10dm}\right)^3 = 0.001m^3$

$V^{1/3} = 0.1m$ take this to be λ

$P = \frac{(1.38\times10^{-23}J/k)(298k)}{\sqrt{2}\pi(1.9\times10^{-10}m)^2(0.1m)} = \boxed{0.26\,Pa = P}$

7. 80% N_2 / 20% O_2

$d(N_2) = 3.8\,\text{Å}$

$d(O_2) = 3.6\,\text{Å}$

σ is from a collision of $1\,N_2 + 1\,O_2$

$$d_{coll} = \frac{d(N_2) + d(O_2)}{2} = \frac{3.8\,\text{Å} + 3.6\,\text{Å}}{2}$$

$d_{coll} = 3.7\,\text{Å}$

$$\sigma = \pi d_{coll}^2 = \underline{4.3\times10^{-19}\,m^2}$$

$$N_D = \frac{N_{TOT}}{V_{TOT}} = \frac{n N_A}{V_{TOT}} \; ; \; V_{TOT} = \frac{nRT}{P} \Rightarrow \underline{N_D = \frac{N_A P}{RT}}$$

$$N_D(N_2) = \frac{N_A(0.8)(P_{TOT})}{RT} = \frac{(6.02\times10^{23}\,/mol)(0.8)P_{TOT}}{(8.02\times10^{-2}\,L\,bar/mol/K)\,T} = 6.0\times10^{24}\,\frac{K}{L\,bar}\left(\frac{P_{TOT}}{T}\right)$$

$$N_D(O_2) = \frac{N_A(0.2)P_{TOT}}{RT} = 1.5\times10^{24}\,K/L\,bar\left(\frac{P_{TOT}}{T}\right)$$

these will change w/altitude

$$Z_{coll} = \sigma c_{rel} N_D(N_2) N_D(O_2) \qquad c_{rel} = \sqrt{2}\,c_{rms} = \sqrt{2}\left(\frac{3RT}{(FW)}\right)^{1/2}$$

FW? use reduced mass: $\frac{m_1 m_2}{m_1+m_2} = \frac{(28\,g/mol)(32\,g/mol)}{28\,g/mol + 32\,g/mol}$

$\mu = 14.9\,g/mol = 0.0149\,kg/mol$

$$\underline{Z_{coll} = \sigma\sqrt{2}\left(\frac{3RT}{\mu}\right)^{1/2} N_D(N_2) N_D(O_2)}$$

now, plug in #'s for each altitude

continued:

@ 20 km: $P_{TOT} = 0.056$ bar, $T = 220$ K

$$Z_{coll} = (4.3\times10^{-19}\,m^2)\sqrt{2}\left(\frac{3(8.315\,J/K\,mol)(220\,K)}{0.0149\,kg/mol}\right)^{1/2}\left(6.0\times10^{24}\frac{K}{L\,bar}\right)\left(\frac{0.056\,bar}{220\,K}\right)$$

$$\times 1.5\times10^{24}\frac{K}{L\,bar}\left(\frac{0.056\,bar}{220\,K}\right)$$

$$Z_{coll} = (4.3\times10^{-19}\,m^2)\sqrt{2}\,(606\,m/s)(1.53\times10^{21}/L)(3.82\times10^{20}/L)$$

$$= 2.2\times10^{26}\,m^3/sL^2\left(\frac{1L}{1dm^3}\right)^2\left(\left(\frac{1dm}{0.1m}\right)^3\right)^2$$

$$\boxed{Z_{coll} = 2.2\times10^{32}\,/sm^3}$$

etc.

8. concentration of H: $[H] = \frac{n_H}{V} = \frac{0.36\rho}{FW} = \frac{(0.36)(158\,g/cm^3)}{1.0\,g/mol}$

$[H] = 57$ mol/cm^3

He: $[He] = \frac{n_{He}}{V} = \frac{(0.64)(158\,g/cm^3)}{4.0\,g/mol}$

$[He] = 25$ mol/cm^3

total $= [H]+[He]+[e^-] = 189$ mol/cm^3

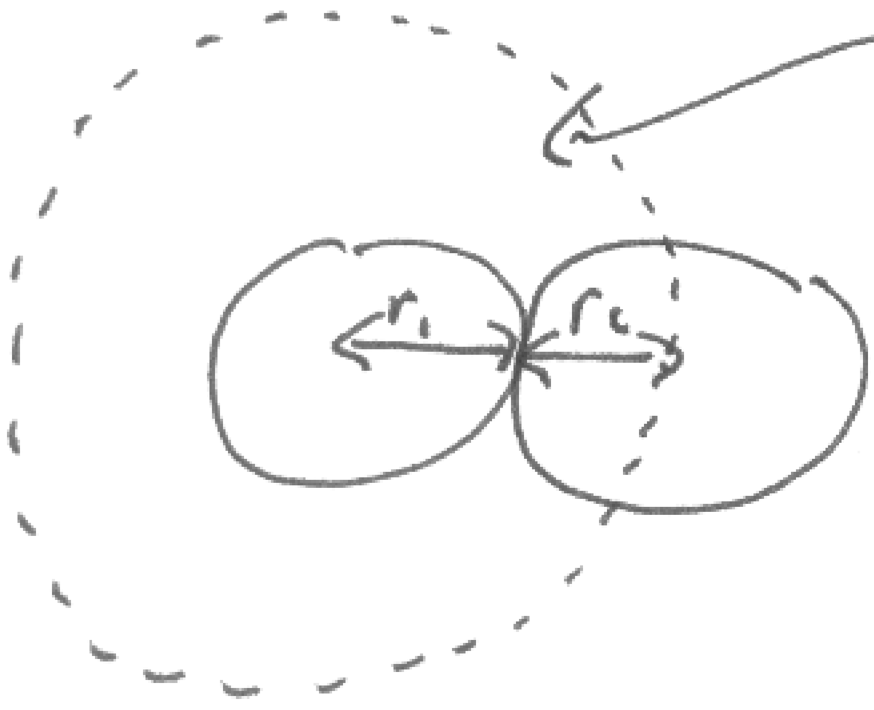
radii: $r_H = (1.4\times10^{-13}\,cm)(1^{1/3}) = 1.4\times10^{-13}\,cm$

$r_{He} = (1.4\times10^{-13}\,cm)(4^{1/3}) = 2.2\times10^{-13}\,cm$

weighted average: $r_{avg} = 0.36(1.4\times10^{-13}\,cm) + (0.64)(2.2\times10^{-13}\,cm) = 1.9\times10^{-13}\,cm$

a) excluded volume

$\boxed{r_{coll} = 2r_{avg}}$



excluded volume of collision pair

$r_{coll} = r_1 + r_2$

excluded volume of a single nucleus =

½ excluded volume of a collision pair

$$V_{ex} = \left(\tfrac{4}{3}\pi r_{coll}^3\right)\left(\tfrac{1}{2}\right)(N_A)$$

collision pair — 1 nucleus — units of mol^{-1}

$$\boxed{V_{ex} = 7.1\times10^{-14}\ cm^3/mol}$$

this is the volume occupied by nuclei per mole of material

b) $V_{ex} << 1\,cm^3$, so ideal gas law ok

$$T = \frac{PV}{nR} = \frac{P}{\frac{n}{V}R}\ ;\ \frac{n}{V} = \text{total concentration}$$

$$T = \frac{(2.5\times10^{11}\,atm)\left(\frac{1\,dm}{10\,cm}\right)^3}{(189\,mol/cm^3)(8.02\times10^{-2}\,L\,atm/mol\,K)\left(\frac{1\,dm^3}{1\,L}\right)}$$

$$\boxed{T = 1.6\times10^{7}\,K}$$