

Problem set #7

$$1. \text{ Bohr model: } E_n = \frac{-Z^2 e^4 m_e}{8 \epsilon_0^2 n^2 h^2} = \frac{-(2.18 \times 10^{-18} \text{ J}) Z^2}{n^2}$$

$$1E: \Delta E = E(n=2) - E(n=1)$$

$$E(n=2) = 0 \text{ always}$$

$$\Delta E = -E(n=1)$$

$$H: Z = +1: \Delta E_1 = - \frac{-(2.18 \times 10^{-18} \text{ J}) (1)^2}{1^2} = 2.18 \times 10^{-18} \frac{\text{J}}{\text{atom}} \left( 6.242 \times 10^{18} \frac{\text{eV}}{\text{J}} \right)$$

$$\boxed{\Delta E(Z=1) = 13.6 \text{ eV/atom} \text{ spot on } 1E_1}$$

$$He^+ Z = +2: \Delta E_1(Z=2) = \frac{(2.18 \times 10^{-18} \text{ J}) (2^2)}{1^2} = 8.72 \times 10^{-18} \frac{\text{J}}{\text{atom}} \left( 6.242 \times 10^{18} \frac{\text{eV}}{\text{J}} \right)$$

$$\boxed{\Delta E(Z=2) = 54.4 \text{ eV/atom} \text{ spot on } 1E_2}$$

$$Li^{2+}: Z = +3 \quad \boxed{\Delta E(Z=3) = 122.5 \text{ eV/atom} \text{ pretty good for } 1E_3}$$

$$Be^{3+}: Z = +4 \quad \boxed{\Delta E(Z=4) = 217.7 \text{ eV/atom} \text{ spot on } 1E_4}$$

$$B^{4+}: Z = +5 \quad \boxed{\Delta E(Z=5) = 340.19 \text{ eV/atom} \text{ pretty good for } 1E_5}$$

So the Bohr model works pretty well for  $1E$ 's of  $1e^-$  atoms.

$$2. \Delta E \Delta t \geq \frac{h}{4\pi}$$

$$\Delta t = 10^{-10} \text{ s}$$

$$n_i = 4$$

$$n_f = 2$$

$$a) \Delta E_{\min} = \frac{h}{4\pi \Delta t} = \frac{(6.626 \times 10^{-34} \text{ J s})}{4\pi (10^{-10} \text{ s})}$$

$$\Delta E_{\min} = 5.27 \times 10^{-25} \text{ J}$$

$$b) \Delta E = h \Delta \nu$$

$$\Delta \nu = \frac{\Delta E}{h} = \frac{5.27 \times 10^{-25} \text{ J}}{6.626 \times 10^{-34} \text{ J s}}$$

$$\Delta \nu = 8.0 \times 10^8 \text{ s}^{-1}$$

$$\Delta E = \frac{hc}{\Delta \lambda}$$

$$\Delta \lambda = \frac{hc}{\Delta E} = \frac{(6.626 \times 10^{-34} \text{ J s})(3.0 \times 10^8 \text{ m/s})}{5.27 \times 10^{-25} \text{ J}}$$

$$\Delta \lambda = 0.38 \text{ m}$$

c) Prediction of the Bohr model:

$$\Delta E = E(n=2) - E(n=4)$$

$$= \frac{-(2.18 \times 10^{-18} \text{ J})(1^2)}{2^2} - \frac{-(2.18 \times 10^{-18} \text{ J})(1^2)}{4^2}$$

\*Note thus  $\Delta E = -5.45 \times 10^{-19} \text{ J} - -1.36 \times 10^{-19} \text{ J}$

" $\Delta E$ " is different  $\Delta E = 4.09 \times 10^{-19} \text{ J}$

$$\lambda = \frac{hc}{\Delta E} = 4.9 \times 10^{-7} \text{ m} = 490 \text{ nm}$$

than in part a). Make sure you understand that.

This implies that the uncertainty in  $\Delta E$  is far larger than  $\Delta E$  itself.

a) This means the emission spectrum of an electron whose lifetime is measured so exactly will be broad, not sharp.

3.  $\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$

$n=2$

$x = \frac{L}{4}$

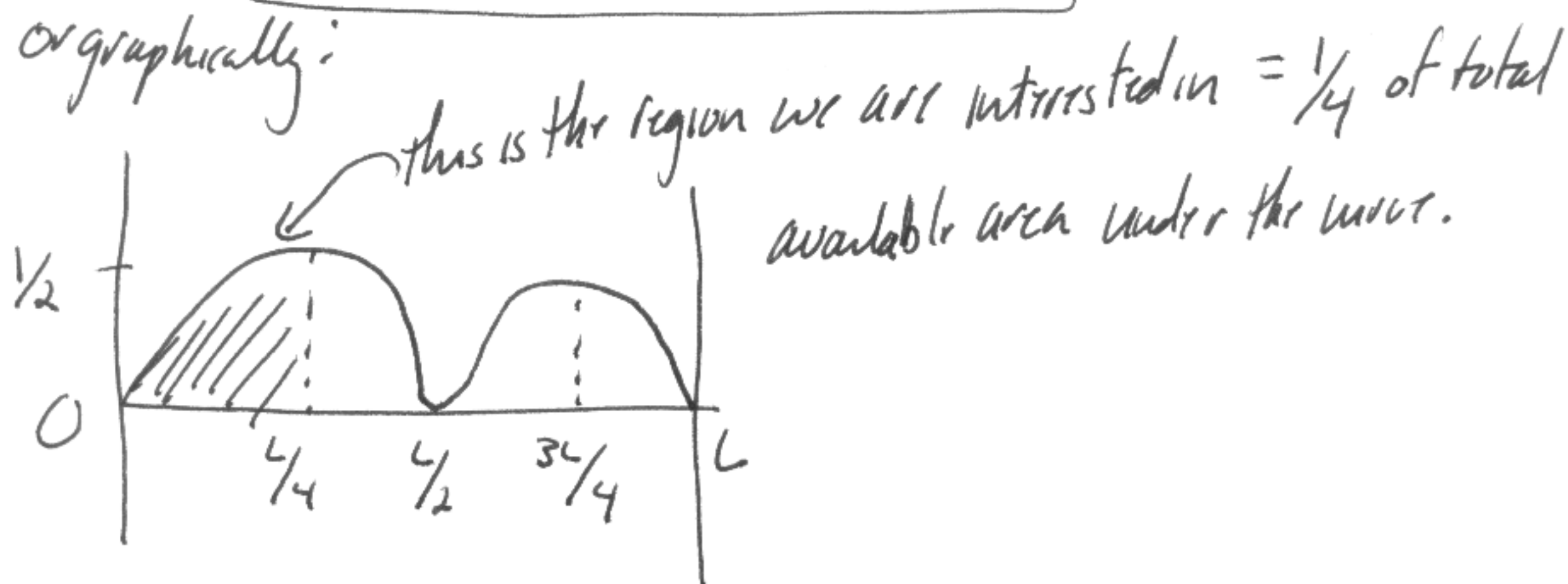
$$P(x) = \psi^2(x) = \frac{2}{L} \sin^2\left(\frac{n\pi x}{L}\right)$$

$$\psi^2\left(\frac{L}{4}\right) = \frac{2}{L} \sin^2\left(\frac{2\pi}{4}\right) = \frac{2}{L} \sin^2\left(\frac{\pi}{2}\right)$$

need to integrate over  $x=0 \rightarrow L/4$

$$\int_0^{L/4} \frac{2}{L} \sin^2\left(\frac{\pi x}{2}\right) dx = \frac{1}{4}$$

or graphically:



4. If the probability of a wavefunction,  $\psi^2(x) = 0$ , means there are regions of space where the particle cannot exist.

No matter how long we look, we will never find the particle there. This is fundamentally new idea from quantum mechanics, with no analogy in classical mechanics.

5.  $L = L$ ,  $L = 1.34 \text{ \AA} = 1.34 \times 10^{-10} \text{ m}$

a)  $E_n = \frac{h^2 n^2}{8mL^2}$

$E_1 = \frac{(6.626 \times 10^{-34} \text{ Js})^2 (1^2)}{8 (9.11 \times 10^{-31} \text{ kg}) (1.34 \times 10^{-10} \text{ m})^2} = \boxed{3.35 \times 10^{-18} \text{ J}}$

$E_2 = 1.34 \times 10^{-17} \text{ J}$

$E_3 = 3.02 \times 10^{-17} \text{ J}$

b)  $n=1 \rightarrow n=2$        $\Delta E = E(n=2) - E(n=1)$

$\Delta E = 1.0 \times 10^{-17} \text{ J}$

$\lambda = \frac{hc}{\Delta E} = \boxed{2.0 \times 10^{-8} \text{ m}}$

6. Need pressure of  $10^{-6}$  atm per  $\text{cm}^{-2}$

→ convert this to force:

$$P = 10^{-6} \text{ atm} \left( \frac{1.01 \times 10^5 \text{ Nm}^{-2}}{\text{atm}} \right) = 1.01 \times 10^{-1} \text{ N/m}^2$$

$$F = PA = (1.01 \times 10^{-1} \text{ N/m}^2) (1 \text{ cm}^2) \left( \frac{1 \text{ m}}{100 \text{ cm}} \right)^2 = 1.01 \times 10^{-5} \frac{\text{kg m}}{\text{s}^2}$$

= Force needed to solve the problem

→ determine momentum of each  $6000 \text{ \AA}$  photon:

$$p = \frac{h}{\lambda} = 1.1 \times 10^{-27} \text{ kg m/s}$$

→ let's assume all momentum is transferred from the photon to the sail elastically.

$$\Delta p = p_2 - p_1 = 2.2 \times 10^{-27} \text{ kg m/s}$$

$$F = \Delta p n \quad (\text{where } n = \# \text{ collisions per unit time})$$

$$n = \frac{F}{\Delta p} = \boxed{4.6 \times 10^{21} \text{ s}^{-1}}$$