CH301H Fall ~~2010~~ 2011

Problem set #7

1. Bohr model: $E_n = \frac{-Z^2 e^4 m_e}{8\epsilon_0^2 n^2 h^2} = \frac{-(2.18\times10^{-18}\,J)Z^2}{n^2}$

IE: $\Delta E = E(n=\infty) - E(n=1)$

$E(n=\infty) = 0$ always

$\Delta E = -E(n=1)$

H: $Z=+1$: $\Delta E_1 = -\frac{-(2.18\times10^{-18}J)(1)^2}{1^2} = 2.18\times10^{-18}\frac{J}{atom}\left(6.242\times10^{18}\frac{eV}{J}\right)$

$\boxed{\Delta E(Z=1) = 13.6\ eV/atom \quad \text{spot on } IE_1}$

He^+ $Z=+2$: $\Delta E_1(Z=2) = \frac{(2.18\times10^{-18}J)(2^2)}{1^2} = 8.72\times10^{-18}\,J/atom\,(6.242\times10^{18}\,eV/J)$

$\boxed{\Delta E(Z=2) = 54.4\ eV/atom \quad \text{spot on } IE_2}$

Li^{2+}: $Z=+3$ $\boxed{\Delta E(Z=3) = 122.5\ eV/atom \quad \text{pretty good for } IE_3}$

Be^{3+}: $Z=+4$ $\boxed{\Delta E(Z=4) = 217.7\ eV/atom \quad \text{spot on } IE_4}$

B^{4+}: $Z=+5$ $\boxed{\Delta E(Z=5) = 340.19\ eV/atom \quad \text{pretty good for } IE_5}$

So the Bohr model works pretty well for IE's of $1e^-$ atoms.

2. $\Delta E \Delta t \geq \frac{h}{4\pi}$

$\Delta t = 10^{-10} s$

$n_i = 4$

$n_f = 2$

a) $\Delta E_{min} = \frac{h}{4\pi \Delta t} = \frac{(6.626\times10^{-34} Js)}{4\pi(10^{-10} s)}$

$\boxed{\Delta E_{min} = 5.27\times10^{-25} J}$

b) $\Delta E = h\Delta\nu$

$\Delta\nu = \frac{\Delta E}{h} = \frac{5.27\times10^{-25} J}{6.626\times10^{-34} Js}$

$\boxed{\Delta\nu = 8.0\times10^{8} s^{-1}}$

$\Delta E = \frac{hc}{\Delta\lambda}$ $\quad \Delta\lambda = \frac{hc}{\Delta E} = \frac{(6.626\times10^{-34} J\cancel{s})(3.0\times10^{8} m/\cancel{s})}{5.27\times10^{-25} \cancel{J}}$

$\boxed{\Delta\lambda = 0.38 m}$

c) Prediction of the Bohr model:

$\Delta E = E(n=2) - E(n=4)$

$= \frac{-(2.18\times10^{-18} J)(1^2)}{2^2} - \frac{-(2.18\times10^{-18} J)(1^2)}{4^2}$

*note this "ΔE" is different than in part a). Make sure you understand that.

$\Delta E = -5.45\times10^{-19} J - -1.36\times10^{-19} J$

$\Delta E = 4.09\times10^{-19} J$

$\lambda = \frac{hc}{\Delta E} = 4.9\times10^{-7} m = 490 nm$

This implies that the uncertainty in ΔE is far larger than ΔE itself.

a) This means the emission spectrum of an electron whose lifetime is measured so exactly will be broad, not sharp.

3. $\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$

$n = 2$

$x = \frac{L}{4}$

$$P(x) = \psi^2(x) = \frac{2}{L} \sin^2\left(\frac{n\pi x}{L}\right)$$

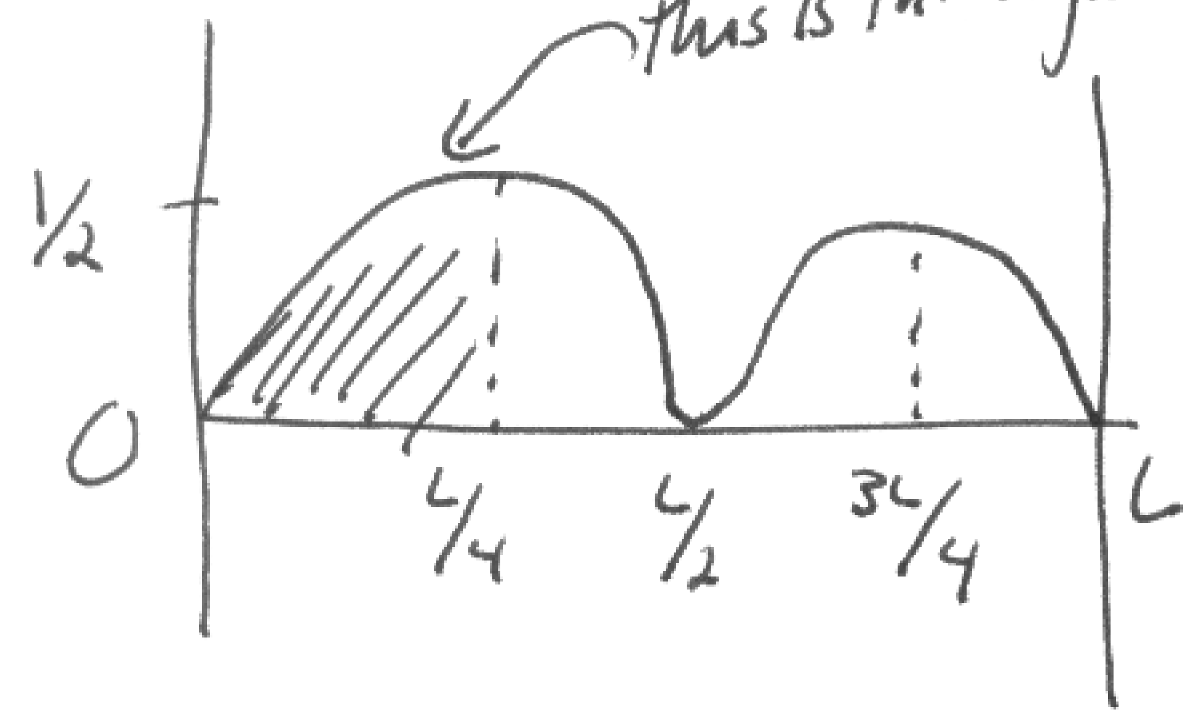
$$\psi^2\left(\frac{L}{4}\right) = \frac{2}{L} \sin^2\left(\frac{2\pi}{4}\right) = \frac{2}{L} \sin^2\left(\frac{\pi}{2}\right)$$

need to integrate over $x = 0 \rightarrow L/4$)

$$\boxed{\int_0^{L/4} \frac{2}{L} \sin^2\left(\frac{\pi}{2}\right) = \frac{1}{4}}$$

or graphically:

this is the region we are interested in $= \frac{1}{4}$ of total available area under the curve.



4. If the probability of a wavefunction, $\psi^2(x) = 0$, means there are regions of space where the particle cannot exist. No matter how long we look, we will never find the particle there. This is fundamentally new idea from quantum mechanics, with no analogy in classical mechanics.

5. C=C, $L = 1.34 \text{Å} = 1.34\times10^{-10}$ m

a) $E_n = \dfrac{h^2 n^2}{8mL^2}$

$$E_1 = \frac{(6.626\times10^{-34}\ \text{Js})^2(1^2)}{8(9.11\times10^{-31}\ \text{kg})(1.34\times10^{-10}\ \text{m})^2} = \boxed{3.35\times10^{-18}\ \text{J}}$$

$\boxed{E_2 = 1.34\times10^{-17}\ \text{J}}$

$\boxed{E_3 = 3.02\times10^{-17}\ \text{J}}$

b) $n=1 \rightarrow n=2$ $\quad \Delta E = E(n=2) - E(n=1)$

$\Delta E = 1.0\times10^{-17}$ J

$$\boxed{\lambda = \frac{hc}{\Delta E} = 2.0\times10^{-8}\ \text{m}}$$

6. Need pressure of 10^{-4} atm per cm^{-2}

→ convert this to force:

$$P = 10^{-4}\,atm\left(\frac{1.01\times10^{5}\ Nm^{-2}}{atm}\right) = 1.01\times10^{-1}\ N/m^2$$

$$F = PA = (1.01\times10^{-1}\ N/m^2)(1cm^2)\left(\frac{1m}{100cm}\right)^2 = 1.01\times10^{-5}\ \frac{kg\,m}{s^2}$$

= Force needed to solve the problem

→ determine momentum of each 6000 Å photon:

$$p = \frac{h}{\lambda} = 1.1\times10^{-27}\ kg\,m/s$$

→ let's assume all momentum is transferred from the photon to the sail elastically.

$$\Delta p = p_2 - p_1 = 2.21\times10^{-27}\ kg\,m/s$$

$F = \Delta p\, n$ (where n = # collisions per unit time)

$$n = \frac{F}{\Delta p} = \boxed{4.6\times10^{21}\ s^{-1}}$$