CH301H: Principles of Chemistry I: Honors
Fall 2015, Unique 49310

Quiz 4, 20 October 2015

The complete wavefunction of an atomic orbital of hydrogen is given below:

$$\psi(r,\theta,\phi) = \frac{2}{81\sqrt{3}}\left(\frac{1}{4\pi}\right)^{1/2}\left(\frac{Z}{a_0}\right)^{3/2}(27-18\sigma+2\sigma^2)\exp\left(\frac{-\sigma}{3}\right)$$

where $\sigma = \dfrac{Zr}{a_0}$

a) How many radial, angular, and total nodes does this orbital have?

b) What are the values of n, l, and m for this orbital?

c) Draw and name the orbital.

d) Extra credit (not much): what values of r give a radial node?

The following information may be useful:

$q = 1.602 \times 10^{-19}$ C	$h = 6.626 \times 10^{-34}$ J s
$\varepsilon_0 = 8.854 \times 10^{-12}$ C^2 J^{-1} m^{-1}	$c = 3.0 \times 10^{8}$ m s^{-1}
$m_e = 9.11 \times 10^{-31}$ kg	$a_0 = 0.529 \times 10^{-10}$ m

a) Θ and Φ do not appear in the wavefunction, so this orbital does not have any angular nodes. By inspection, $\psi = 0$ when σ = large (i.e. high value of r)! →

Quiz 4 Key continued

and when $(27-18\sigma+2\sigma^2)=0$. Even if you cannot solve this polynomial easily, you should see by inspection that it is 0 in two places. Therefore, this wave function has two radial nodes.

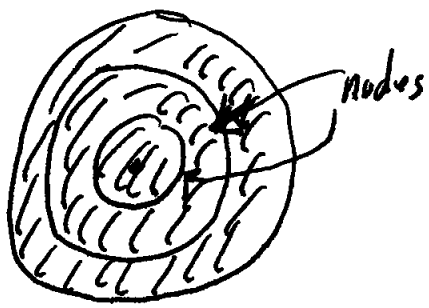
Total nodes = radial nodes + angular nodes = $2+0=2$

b) From part a). n = total nodes + 1 = 3

l = angular nodes = 0

$m=0$

c) This is a 3s orbital:



d) $27-18\sigma+2\sigma^2=0$

$13.5-9\sigma+\sigma^2=0$

$-9\sigma+\sigma^2=-13.5$

add $\left(\frac{9}{2}\right)^2$ to both sides:

$20.25-9\sigma+\sigma^2=6.75$

factor:

$(\sigma+-4.5)^2=6.75$

$\sigma+-4.5=\pm 2.598$

① $\sigma+-4.5=2.598$

$\sigma=7.098=\frac{(1)r}{a_0}$

$\boxed{r=7.1a_0}$

② $\sigma+-4.5=-2.598$

$\sigma=1.9=\frac{(1)r}{a_0}$

$\boxed{\sigma=1.9a_0}$