CH353 Spring 2012

HW3 Key

1. $P_{ext} = 1.0\,atm\left(\frac{1.013\times10^5\,Pa}{atm}\right) = 1.013\times10^5\,Pa$

$A = 1m^2$

$h = 500cm\left(\frac{1m}{100cm}\right) = 5m$

$w = ?$

$w = -P_{ext}\Delta V = -P_{ext}A\Delta h$

$w = -(1.013\times10^5\,Pa)(1m^2)(5m)$

$\boxed{w = -5.1\times10^5\,J}$ (note that $1J = 1\,Pa\,m^3$)

2. $n = 2.00mol$

$T = 0°C = 273K$, $\Delta T = 0$ (isothermal)

$V_i = 1L$

$V_f = 5L$

a) reversible: $\Delta U = \Delta H = 0$ (because isothermal)

$w = -nRT\ln\left(\frac{V_f}{V_i}\right) = -(2.00mol)(8.314\,J/Kmol)(273K)\ln\left(\frac{5L}{1L}\right)$

$\boxed{w = -7.3\times10^3\,J}$

$q = \Delta U - w = -w$; $\boxed{q = 7.3\times10^3\,J}$

b) $P_{ext} = P_f$ $\boxed{\Delta H = \Delta U = 0}$

$$w = -P_{ext}\Delta V = \frac{nRT}{V_f}\Delta V = \frac{(2.00\,mol)(8.02\times10^{-2}\,\frac{L\,atm}{mol\,K})(273\,K)(4\,L)}{5\,L}$$

$$w = -35\,atm\,L\left(\frac{1.01\times10^5\,Pa}{1\,atm}\right)\left(\frac{1\,dm^3}{1\,L}\right)\left(\frac{1\,m}{10\,dm}\right)^3$$

$\boxed{w = -3.63\times10^3\,J}$ $\boxed{q = 3.63\times10^3\,J}$

c) Free expansion: $P_{ext} = 0$ $\boxed{\Delta U = \Delta H = q = w = 0}$

3. $P(V_m - b) = RT$ adiabatic expansion $\Rightarrow dq = 0$

$$dU = dw = C_v\,dT \;;\; dw = -P\,dV_m \;;\; P = \frac{RT}{V_m - b}$$

$$dw = -\frac{RT}{V_m - b}\,dV_m$$

$$dU = dw;\quad C_{v,m}\,dT = -\frac{RT}{V_m - b}\,dV_m$$

$$\int_{T_i}^{T_f} \frac{C_{v,m}\,dT}{T} = -R\int_{V_i}^{V_f} \frac{1}{V_m - b}\,dV_m$$

$$C_{v,m}\ln\left(\frac{T_f}{T_i}\right) = -R\ln\left(\frac{V_{m,f} - b}{V_{m,i} - b}\right)$$

$$\ln\left(\frac{T_f}{T_i}\right)^{C_{vm}/R} = \ln\left(\frac{V_{m,i}-b}{V_{m,f}-b}\right)$$

monatomic gas $\Rightarrow$ $C_{vm} = \frac{3}{2}R$

$$\left(\frac{T_f}{T_i}\right)^{3/2} = \frac{V_{m,i}-b}{V_{m,f}-b}$$

4. monatomic ideal gas $\Rightarrow$ $C_{vm} = \frac{3}{2}R$

$T_i = 300K$

$T_f = 400K$

$P_i = 1.0 atm$

constant volume $\Rightarrow$ $\boxed{w=0}$

$\Delta U = nC_{vm}\Delta T = (1.0 mol)(3/2)(8.314 J/K mol)(100K)$

$\boxed{\Delta U = 1.25 kJ}$

$\boxed{q = \Delta U - w = 1.25 kJ}$

5. $V_i = 22.7 L/mol$

$P_i = 1 bar$

$T_i = 273K$

$V_f = 45.4 L/mol$

$T_f = ?$

monatomic ideal gas $\Rightarrow$ $C_{vm} = \frac{3}{2}R$

$C_{pm} = C_{vm} + R = \frac{5}{2}R$

$$\frac{T_f}{T_i} = \left(\frac{V_f}{V_i}\right)^{\gamma-1} \qquad \gamma = \frac{5}{3}$$

$\boxed{T_f = 172.1K}$

6. $q_p = 4 \frac{kJ}{kg\,h}$

$m = 65\ kg$

$C_{pm}(H_2O) = 75.29\ J/mol\,K = 0.07529\ kJ/mol\,K$

assume body is 100 % H_2O: $FW(H_2O) = 18.015\ g/mol = 0.018015\ \frac{kg}{mol}$

$$n = \frac{65\ kg}{0.018015\ kg/mol} = 3608\ mol$$

$\Delta H = q_p = n C_{pm} \Delta T$

$$\Delta T = \frac{q_p}{n C_{pm}} = \frac{4\ kJ/kg\,h}{(3608\ mol)(0.07529\ kJ/mol\,K)}$$

$\Delta T = 0.0147\ K/kg\,h$

So 65 kg heats up at rate $(0.0147\ K/kg\,h)(65\ kg) = \boxed{0.956\ K/h}$

You probably should not do this for > 1 hr.