HW 3 Key

1) See HW2 key

2) $n = 2.00 \text{ mol}$

$T = 0°C = 273 K$, $\Delta T = 0$ (isothermal)

$V_i = 1 L$

$V_f = 5 L$

a) reversible: $\boxed{\Delta U = \Delta H = 0}$ (isothermal)

$$w = -nRT\ln(V_f/V_i) = -(2.00 \text{ mol})(8.314 \text{ J/mol K})(273 \text{ K})\ln\left(\frac{5L}{1L}\right)$$

$$\boxed{w = -7.3 \times 10^3 \text{ J}}$$

$$q = \Delta U - w = -w \Rightarrow \boxed{q = 7.3 \times 10^3 \text{ J}}$$

b) $P_{ext} = P_f$ $\boxed{\Delta H = \Delta U = 0}$

$$w = -P_{ext}\Delta V = \frac{nRT}{V_f}\Delta V = \frac{(2.00 \text{ mol})(8.02 \times 10^{-2} \frac{L \cdot atm}{mol\ K})(273 \text{ K})}{5L}(4L)$$

$$w = \frac{-35 \text{ L}\cdot\text{atm}}{1}\left|\frac{101325 \text{ Pa}}{1 \text{ atm}}\right|\frac{1 \text{ dm}^3}{1 L}\left|\left(\frac{1 \text{ m}}{10 \text{ dm}}\right)^3\right.$$

$$\boxed{w = -3.63 \times 10^3 \text{ J} \Rightarrow q = 3.63 \times 10^3 \text{ J}}$$

c) Free Expansion: $P_{ext} = 0$

$$\boxed{\Delta U = \Delta H = q = w = 0}$$

3) $P(V_m - b) = RT$ adiabatic expansion $\Rightarrow dq = 0$

$$dU = dw = C_V dT \qquad dw = -P\,dV_m \qquad P = \frac{RT}{V_m - b}$$

$$dw = \frac{-RT}{V_m - b}\,dV_m$$

$$dU = dw\ ; \quad C_{V,m}\,dT = \frac{-RT}{V_m - b}\,dV_m$$

$$\int_{T_i}^{T_f} \frac{C_{V,m}\,dT}{T} = -R\int_{V_i}^{V_f} \frac{1}{V_m - b}\,dV_m$$

$$C_{V,m}\ln\left(T_f/T_i\right) = -R\ln\left(\frac{V_{mf} - b}{V_{mi} - b}\right)$$

$$\ln\left(T_f/T_i\right)^{C_{V,m}/R} = \ln\left(\frac{V_{mi} - b}{V_{mf} - b}\right)$$

For a monatomic gas, $C_{V,m} = \frac{3}{2}R$

$$\boxed{\left(T_f/T_i\right)^{3/2} = \frac{V_{mi} - b}{V_{mf} - b}}$$

4) monatomic ideal gas $\Rightarrow C_{V,m} = \frac{3}{2}R$

$T_i = 300\ K$

$T_f = 400\ K$

$P_i = 1.0\ atm$

constant volume $\Rightarrow \boxed{w = 0}$

$$\Delta U = n C_{V,m}\Delta T = (1.0\ mol)(\tfrac{3}{2})(8.314\ J/mol\ K)(100\ K)$$

$$\boxed{\Delta U = 1.25\ kJ}$$

$$\boxed{q = \Delta U - w = 1.25\ kJ}$$

5) $V_i = 22.7\ L/mol$

$P_i = 1\ bar$

$T_i = 273\ K$

$V_f = 45.4\ L/mol$

$T_f = ?$

monatomic ideal gas $\Rightarrow C_{V,m} = \frac{3}{2}R$

$C_{p,m} = C_{V,m} + R = \frac{5}{2}R$

$$T_f/T_i = \left(\frac{V_f}{V_i}\right)^{\gamma - 1} \qquad \gamma = 5/3$$

$$\boxed{T_f = 172.1\ K}$$

6) $q_p = 4\ kJ/kg\,h$ $\quad m = 65\ kg$ $\quad C_{pm}(H_2O) = 75.29\ J/mol\,K = 0.07529\ kJ/mol\,K$

Assume body is 100% H_2O: $FW(H_2O) = 18.015\ g/mol = 0.018015\ kg/mol$

$$n = \frac{65\ kg}{0.018015\ kg/mol} = 3608\ mol$$

$$\Delta H = q_p = n\,C_{p,m}\Delta T$$

$$\Delta T = \frac{q_p}{n\,C_{p,m}} = \frac{4\ kJ/kg\,h}{(3608\ mol)(0.07529\ kJ/mol\,K)}$$

$$\Delta T = 0.0147\ K/kg\cdot h$$

So, 65 kg heats @ rate $(0.0147\ K/kg\,h)(65\ kg) = 0.956\ K/h$

You probably shouldn't do this for > 1 hour