CH353 – Physical Chemistry I
Spring 2013, Unique 52575

Quiz 3, 22 February 2013

A 45.0 kg ingot of pure iron at 1200 K is completely immersed in a 100 L vat of pure water initially at room temperature until thermal equilibrium is reached. If there is no heat exchange between the water and the surroundings, determine the final temperature of the system, ΔS_{sys}, ΔS_{surr}, and ΔS_{tot}. The following information may be useful.

$T_i(H_2O) = 298\,K$

$q_{Fe} + q_{H_2O} = 0$

	ΔH_f^0 (kJ mol^{-1})	S_m^0 (J K^{-1} mol^{-1})	C_P (J K^{-1} mol^{-1})	ρ (g cm^{-3})
Fe(s)	0	27.3	25.1	7.87
H_2O(l)	-285.8	69.9	75.3	~~0.997~~ 1.00
N_2(g)	0	191.6	29.1	0.880
O_2(g)	0	205.1	29.4	1.14

$FW(Fe) = 55.8\ g/mol$; $FW(H_2O) = 18\ g/mol$

$$n(Fe) = \frac{45000\,g}{55.8\,g/mol} = 806.5\,mol$$

$$n(H_2O) = 100\,L\left(\frac{1\,dm^3}{1\,L}\right)\left(\frac{10\,cm}{1\,dm}\right)^3\left(\frac{1.00\,g}{1\,cm^3}\right)\left(\frac{1\,mol}{18\,g}\right) = 5555\,mol$$

$$n(Fe)C_p(Fe)(T_f - T_i(Fe)) + n(H_2O)C_p(H_2O)(T_f - T_i(H_2O)) = 0$$

Solve for T_f: $n(Fe)C_p(Fe)T_f - n(Fe)C_p(Fe)T_i(Fe) + n(H_2O)C_p(H_2O)T_f - n(H_2O)C_p(H_2O)T_i(H_2O) = 0$

$$n(Fe)C_p(Fe)T_f + n(H_2O)C_p(H_2O)T_f = n(Fe)C_p(Fe)T_i(Fe) + n(H_2O)C_p(H_2O)T_i(H_2O)$$

$$T_f = \frac{n(Fe)C_p(Fe)T_i(Fe) + n(H_2O)C_p(H_2O)T_i(H_2O)}{n(Fe)C_p(Fe) + n(H_2O)C_p(H_2O)}$$

$$T_f = \frac{(806.5\,mol)(25.1\,J/K\,mol)(1200\,K) + (5555\,mol)(75.3\,J/K\,mol)(298\,K)}{(806.5\,mol)(25.1\,J/K\,mol) + (5555\,mol)(75.3\,J/K\,mol)}$$

$T_f = 340\,K$

$T_f > T_i(H_2O)$, $T_f < T_i(Fe)$, so makes sense.

quiz 3 continued

$$\Delta S_{sys} = \Delta S(Fe) + \Delta S(H_2O) = n(Fe)\, C_p(Fe) \ln\left(\frac{T_f}{T_i(Fe)}\right) + n(H_2O)\, C_p(H_2O) \ln\left(\frac{T_f}{T_i(H_2O)}\right)$$

$$\Delta S_{sys} = (806.5\text{ mol})(25.1\text{ J/K mol}) \ln\left(\frac{340\text{ K}}{1200\text{ K}}\right) + (5555\text{ mol})(75.3\text{ J/K mol}) \cdot \ln\left(\frac{340\text{ K}}{298\text{ K}}\right)$$

$$\boxed{\Delta S_{sys} = 29.6\text{ kJ/K}}$$

$\Delta S_{surr} = 0$ (because adiabatic, $q_{surr} = 0$)

$$\boxed{\Delta S_{TOT} = \Delta S_{sys} + \Delta S_{surr} = 29.6\text{ kJ/K}}$$