

## HW 3 Key

1) See HW2 key

2)  $n = 2.00 \text{ mol}$

$T = 0^\circ\text{C} = 273\text{K}$ ,  $\Delta T = 0$  (isothermal)

$V_i = 1\text{L}$

$V_f = 5\text{L}$

a) reversible:  $\Delta U = \Delta H = 0$  (isothermal)

$$w = -nRT \ln\left(\frac{V_f}{V_i}\right) = -(2.00 \text{ mol})(8.314 \frac{\text{J}}{\text{mol K}})(273 \text{ K}) \ln\left(\frac{5\text{L}}{1\text{L}}\right)$$

$$w = -7.3 \times 10^3 \text{ J}$$

$$q = \Delta U - w = -w \Rightarrow q = 7.3 \times 10^3 \text{ J}$$

b)  $P_{\text{ext}} = P_f$   $\Delta H = \Delta U = 0$

$$w = -P_{\text{ext}} \Delta V = \frac{nRT}{V_f} \Delta V = \frac{(2.00 \text{ mol})(8.02 \times 10^{-2} \frac{\text{L atm}}{\text{mol K}})(273 \text{ K})}{5\text{L}} (4\text{L})$$

$$w = \frac{-35 \text{ L atm}}{1} \left| \frac{101325 \text{ Pa}}{1 \text{ atm}} \right| \left| \frac{1 \text{ dm}^3}{1 \text{ L}} \right| \left| \frac{1 \text{ m}}{10 \text{ dm}} \right|^3$$

$$w = -3.63 \times 10^3 \text{ J} \Rightarrow q = 3.63 \times 10^3 \text{ J}$$

c) Free Expansion:  $P_{\text{ext}} = 0$

$$\Delta U = \Delta H = q = w = 0$$

$$3) P(V_m - b) = RT \quad \text{adiabatic expansion} \Rightarrow dq = 0$$

$$dU = dw = C_{v,m} dT \quad dw = -P dV_m \quad P = \frac{RT}{V_m - b}$$

$$dw = \frac{-RT}{V_m - b} dV_m$$

$$dU = dw; \quad C_{v,m} dT = \frac{-RT}{V_m - b} dV_m$$

$$\int_{T_i}^{T_f} \frac{C_{v,m} dT}{T} = -R \int_{V_i}^{V_f} \frac{1}{V_m - b} dV_m$$

$$C_{v,m} \ln\left(\frac{T_f}{T_i}\right) = -R \ln\left(\frac{V_{mf} - b}{V_{mi} - b}\right)$$

$$\ln\left(\frac{T_f}{T_i}\right)^{C_{v,m}/R} = \ln\left(\frac{V_{mi} - b}{V_{mf} - b}\right)$$

For a monatomic gas,  $C_{v,m} = \frac{3}{2}R$

$$\boxed{\left(\frac{T_f}{T_i}\right)^{3/2} = \frac{V_{mi} - b}{V_{mf} - b}}$$

4) monatomic ideal gas  $\Rightarrow C_{v,m} = \frac{3}{2}R$

$$T_i = 300 \text{ K}$$

$$T_f = 400 \text{ K}$$

$$P_i = 1.0 \text{ atm}$$

constant volume  $\Rightarrow \boxed{w = 0}$

$$\Delta U = n C_{v,m} \Delta T = (1.0 \text{ mol}) \left(\frac{3}{2}\right) (8.314 \text{ J/mol K}) (100 \text{ K})$$

$$\boxed{\Delta U = 1.25 \text{ kJ}}$$

$$\boxed{q = \Delta U - w = 1.25 \text{ kJ}}$$

$$5) V_i = 22.7 \text{ L/mol}$$

$$P_i = 1 \text{ bar}$$

$$T_i = 273 \text{ K}$$

$$V_f = 45.4 \text{ L/mol}$$

$$T_f = ?$$

monatomic ideal gas  $\Rightarrow C_{v,m} = \frac{3}{2}R$

$$C_{p,m} = C_{v,m} + R = \frac{5}{2}R$$

$$\frac{T_f}{T_i} = \left(\frac{V_f}{V_i}\right)^{\gamma-1}$$

$$\gamma = \frac{5}{3}$$

$$\boxed{T_f = 172.1 \text{ K}}$$

$$6) \quad q_p = 4 \frac{\text{kJ}}{\text{kg}\cdot\text{h}} \quad m = 65 \text{ kg} \quad C_{pm}(\text{H}_2\text{O}) = 75.29 \frac{\text{J}}{\text{mol}\cdot\text{K}} = 0.07529 \frac{\text{kJ}}{\text{mol}\cdot\text{K}}$$

Assume body is 100%  $\text{H}_2\text{O}$ :  $\text{FW}(\text{H}_2\text{O}) = 18.015 \frac{\text{kg}}{\text{mol}} = 0.018015 \frac{\text{kg}}{\text{mol}}$

$$n = \frac{65 \text{ kg}}{0.018015 \frac{\text{kg}}{\text{mol}}} = 3608 \text{ mol}$$

$$\Delta H = q_p = n C_{pm} \Delta T$$

$$\Delta T = \frac{q_p}{n C_{pm}} = \frac{4 \frac{\text{kJ}}{\text{kg}\cdot\text{h}}}{(3608 \text{ mol})(0.07529 \frac{\text{kJ}}{\text{mol}\cdot\text{K}})}$$

$$\Delta T = 0.0147 \frac{\text{K}}{\text{kg}\cdot\text{h}}$$

$$\text{So, } 65 \text{ kg heats @ rate } (0.0147 \frac{\text{K}}{\text{kg}\cdot\text{h}})(65 \text{ kg}) = 0.956 \frac{\text{K}}{\text{h}}$$

You probably shouldn't do this for  $> 1$  hour