Particle-in-a-box:

Schrodinger Equation: $$\frac{-h^2}{8\pi^2 m}\frac{d^2\psi(x)}{dx^2} = E\psi(x)$$

Acceptable wavefunction: $$\psi(x) = A\sin\left(\frac{n\pi x}{L}\right)$$

Normalize: $$\int_0^L \psi^2 dx = \int_0^L A^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

Solve for *A*: $$\int_0^L \sin^2\left(\frac{n\pi x}{L}\right) dx = \frac{L}{2}$$

$$A^2\left(\frac{L}{2}\right) = 1 \qquad A = \sqrt{\frac{2}{L}}$$

Exact wavefunction: $$\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$$

Schrodinger Equation: $$\frac{d^2\psi(x)}{dx^2} = \frac{-8\pi^2 m}{h^2}E\psi(x)$$

Left-hand side… $$\frac{d^2\psi(x)}{dx^2} = -\left(\frac{n\pi}{L}\right)^2\psi(x)$$

…must equal right-hand side: $$-\left(\frac{n\pi}{L}\right)^2\psi(x) = \frac{-8\pi^2 m}{h^2}E\psi(x)$$

Solve for *E*: $$E = \frac{n^2h^2}{8mL^2}$$